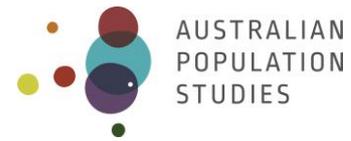


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# Introductory Guide



## A brief guide to producing a national population projection

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Paper received 10 February 2021; accepted 23 March 2021; published 31 May 2021

### Abstract

#### Background

There are surprisingly few resources available which offer an introductory guide to preparing a national population projection using a cohort-component model. Many demography textbooks cover projections quite briefly, and many academic papers on projections focus on advanced technical issues.

#### Aims

The aim of this paper is to provide a short and accessible guide to producing a national-scale population projection using the cohort-component model.

#### Data and methods

The paper describes the cohort-component model from a population accounting perspective, presents all the necessary projection calculations, and covers the key steps which form part of the projections preparation process – from gathering input data to validating outputs. An accompanying Excel workbook implements the model and contains example projections for Australia.

#### Conclusions

Calculating a national population projection using a cohort-component model involves fairly simple algebra, but the broader projections preparation process is more complex, and requires careful consideration and judgement.

#### Key words

Population projections; cohort-component model; population accounting; Australia

## 1. Introduction

National statistical offices and some international agencies, such as the United Nations, produce national population projections on a regular basis. Projections are used for many purposes, including calculating the amount of additional housing needed in coming years, estimating future aged care demand, assessing the fiscal implications for government of future population growth and ageing (e.g., Australian Government 2015), and contributing to national debates about population change. Researchers also produce national population projections from time to time in order to understand demographic change, explore alternative demographic scenarios, or demonstrate methodological refinements, among other reasons (e.g., Lomax et al. 2019; Wilson et al. 2021).

Surprisingly, the guidance available on how to create national population projections is quite limited. Many academic papers and books on projections, as would be expected, report the latest research and usually focus on advanced technical aspects of projections. There are relatively few handbooks or websites which provide introductory advice on how to create population projections. Examples of resources which provide guidance include a manual on projections published by the United Nations (1989), the UNFPA/IUSSP web-based demography course which includes a section on projections (Timaeus 2015), and manuals for specific population projection software products (e.g., Shorter et al. 1995; Stover and Kirmeyer 2007; US Census Bureau 2019); and the *Practitioner's Guide* by Smith et al. (2013) is a good example for subnational projections. Most demography textbooks do include a section on the principal model used to create national population projections – the cohort-component model – though the description is often brief. And the model usually includes only net international migration (which has some limitations) rather than separate immigration and emigration flows.

The aim of this paper is to present a short practical guide to producing a national population projection. Our goal is to offer an accessible introduction to projection modelling which avoids complex algebra but includes sufficient mathematical detail so that readers can produce their own projections if desired. We present the cohort-component model from a population accounting perspective because it offers an intuitive and transparent way of expressing the model. This approach is based on the well-known population accounting (or balancing) equation, and involves projecting the demographic components of change as the product of occurrence/exposure rates and populations-at-risk. Our description is less directly connected to the life table than some other presentations of the cohort-component model, and we do not express the model in matrix algebra as many researchers do<sup>1</sup>. Our projection equations also differ somewhat from those used by the Australian Bureau of Statistics for their population projections (ABS 1999). Primarily, the description in this paper is most relevant to countries with good statistical systems and reasonable quality demographic data.

We begin with a few preliminary terms and concepts (section 2) and then give an overview of the projections process (section 3). The cohort-component model forms the focus of section 4, while section 5 covers the preparation of necessary input data and assumptions. In section 6 we briefly describe how projection outputs can be checked for plausibility and consistency, before a few

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<sup>1</sup> Readers interested in matrix arrangements of the cohort-component model are recommended to consult Keyfitz and Caswell (2005: chapter 3) or Rogers (1995: chapter 3).

concluding remarks are given in section 7. Accompanying this paper is an Excel workbook which incorporates a cohort-component model for national population projections, illustrated with example projections for Australia. It is available from <https://doi.org/10.6084/m9.figshare.13565222.v3>.

## 2. Preliminaries

Before preparing any national-scale population projections, it is very useful to have a reasonable knowledge of:

- projections terminology,
- life tables,
- the Lexis diagram, and
- population accounting.

We also assume knowledge of basic demographic concepts and terms, which are usually covered in the initial chapters of introductory demography textbooks (e.g., Preston et al. 2001; Rowland 2003).

### 2.1. Projections terminology

Box 1 defines terms population projections terms used in this paper.

#### Box 1: Definitions

**Jump-off year:** the year from which the projections ‘jump off’ into the demographic future. It is sometimes also known as the launch year.

**Population estimate:** the population for a past point in time based on information available for that time.

**Population projection:** a calculation of population beyond the jump-off year based on selected assumptions about the future drivers of population change (commonly, fertility, mortality and migration).

**Population forecast:** a population projection regarded as the most likely outcome; a prediction.

**Projection horizon:** the time period between the jump-off year and the final year of the projections.

**Projection interval:** the time interval between each projected population (usually 1 or 5 years).

**Projection assumptions:** data inputs to the projection calculations which usually include the assumed future paths of fertility, mortality, and migration (and sometimes other variables). More broadly defined, projection assumptions also implicitly or explicitly include qualitative elements, such as expectations about future immigration policy.

**Population-at-risk:** The number of person-years lived in a projection interval by a particular population exposed to the ‘risk’ of a demographic event. It is often approximated as the mean of the start and end populations of a projection interval.

**(Occurrence/exposure) rate:** The ratio of the number of demographic events occurring divided by the population exposed to the risk of that event (population-at-risk).

Source: Wilson (2019); authors’ definitions

## 2.2. The life table

A life table is a model of a cohort's mortality experience over its lifetime, but it is often calculated for a particular year or group of years and therefore describes a synthetic cohort. Although we do not make extensive use of life table measures in this paper, it is helpful to understand the calculations behind, and meaning of,  ${}_nL_x$ , the life table stationary population for each age group. In life table notation,  $x$  is the exact starting age of an age group and  $n$  is the width of the age group in years. The life table is covered comprehensively in demography textbooks, so is not described here. Readers unfamiliar with the life table are recommended to consult an appropriate source (e.g. Kinter 2004; Preston et al. 2001: chapter 3; Rowland 2003: chapter 8; Sloggett 2015). We return to life tables in section 5 on input data preparation.

## 2.3. The Lexis diagram (or age-time plan)

The Lexis diagram consists of a graph with age increasing up the vertical axis and time progressing along the horizontal axis. It provides a very useful way of precisely locating the age, time and cohort extent of demographic stocks and flows in population projections (and, more generally, any demographic analysis). Longer discussions on the Lexis diagram can be found in Calot (2015), Pressat (2008: chapter 2), and Rau (2018).

Figure 1 illustrates some key demographic variables. If the Lexis diagram is used to illustrate the demographic history of just a handful of individuals then individual lifelines can be drawn, and demographic events (e.g. death, migration, giving birth) marked at points on those lines (Figure 1, upper diagram). An individual's lifeline starts on the horizontal axis and proceeds at a 45° angle because as each year passes, the person ages by 1 year. Symbols are placed on lifelines to denote demographic events, such as giving birth, migration, and eventually, death.

However, once there are more than just a few lifelines the diagram becomes cluttered, and it becomes necessary to use thick lines to denote all the lifelines crossing a point in time (or exact age) and shapes to represent all demographic events occurring within a particular age-time space. For example, in the lower diagram in Figure 1 the thick vertical lines represent the population of the cohort aged 1 year old last birthday at time  $t$  and aged 2 years at time  $t + 1$ . The parallelogram shaded grey defines the period-cohort age-time space which refers to demographic events experienced by the cohort over the one year period. The shaded square depicts demographic events defined by period and age. The thick lines at the left and right sides of the square represent populations of that age at the start and end of the time interval. The horizontal thick blue line denotes babies born during a year, and the shaded triangle is the age-period-cohort space representing demographic events occurring to this cohort of newly-born babies in that year.

## 2.4. Population accounting

Population accounts connect population stocks at two points in time with the demographic components which add to, and deplete, the population (Rees 1985; Rees and Willekens 1986). At the national scale, the population at time  $t + 1$  equals the population at time  $t$  plus births, minus deaths, plus immigration, minus emigration:

$$P(t + 1) = P(t) + B(t, t + 1) - D(t, t + 1) + I(t, t + 1) - E(t, t + 1) \quad (1)$$

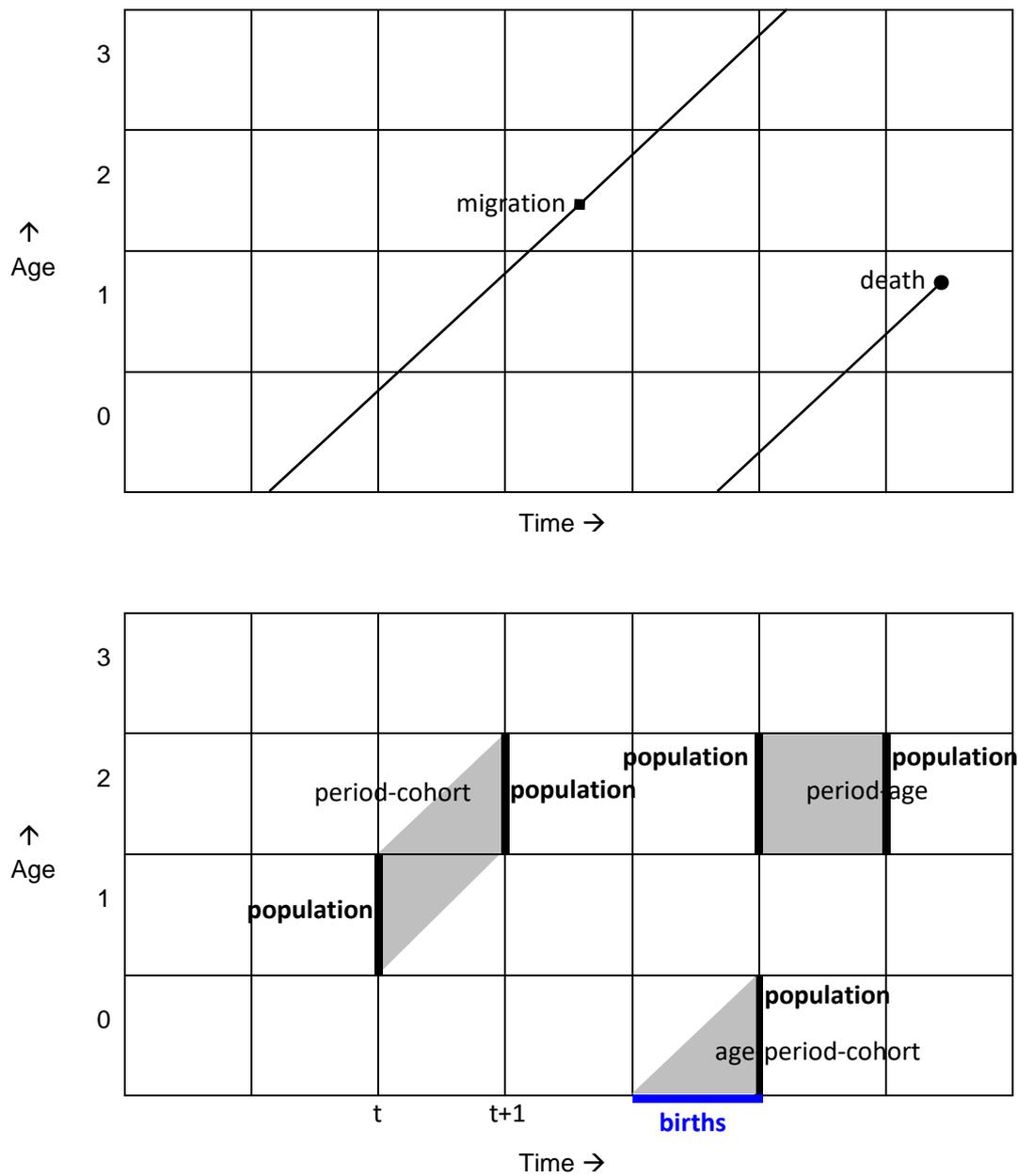


Figure 1: The age-time location of lifelines and demographic stocks and flows

where:

$P$  = population

$t$  = a point in time

$t + 1$  = a point in time one year later

$t, t + 1$  = the one year interval starting at  $t$  and ending at  $t+1$

$B$  = births

$D$  = deaths

$I$  = immigration

$E$  = emigration

This is the demographic accounting, or balancing, equation for a national population. Strictly, it refers to what are known as *movement* population accounts in which the migration flows are counts of moves across the national border. An alternative population accounting equation in which the migration components refer to counts of migrants (not moves) uses data on people living in different countries at time  $t$  and time  $t + 1$  (as measured by a census). This *transition* accounts perspective is more complex and is not considered in this paper<sup>2</sup>.

As an example, consider the population accounts for Australia over the 12 month period from mid-2019 to mid-2020 (ABS 2020a). The start-of-interval population  $P(t)$  was 25,365,745, the number of births 304,101, deaths 166,974, immigration 486,757, and emigration 302,588. The end-of-interval population  $P(t + 1)$  was therefore:

$$\begin{aligned} &= 25,365,745 + 304,101 - 166,974 + 486,757 - 302,588 \\ &= 25,687,041. \end{aligned}$$

### 3. The process for producing a national population projection

The population projections process involves a number of steps, starting with gathering data, checking data and making adjustments if necessary, preparing projection assumptions, entering data into your projection program, calculating the projections, and then checking outputs.

*Step 1: Obtain the jump-off populations and all data needed to prepare fertility, mortality and migration assumptions*

All projections begin with the jump-off population, which is usually the most up-to-date set of population estimates published by the national statistical office. At the national scale, projections of fertility and mortality are often created by models which work best with many decades of data. The extent of historical time series required will depend on the precise fertility and mortality projection approaches chosen (discussed in section 5 later). Immigration and emigration data are also ideally obtained for the last few decades – or at the very least total immigration and emigration flows for several decades, and by age and sex for the most recent years. All projections begin with the jump-off population, which is usually the most up-to-date set of population estimates published by the national statistical office. At the national scale, projections of fertility and mortality are often created by models which work best with many decades of data. The extent of historical time series required will depend on the precise fertility and mortality projection approaches chosen (discussed in section 5 later). Immigration and emigration data are also ideally obtained for the last few decades – or at the very least total immigration and emigration flows for several decades, and by age and sex for the most recent years.

*Step 2: Check that the input data are good quality and that recent population accounts are consistent, and carry out adjustments if necessary*

Past fertility and mortality rates by age are usefully graphed to check for any obvious data problems. The national statistical office which published the data may also have accompanying notes or reports on data quality. In most countries with well-developed statistical systems, the quality and coverage

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<sup>2</sup> Interested readers are recommended to consult Rees (1986) or Rees and Wilson (1977).

of births and deaths data are normally good (though not perfect). In Australia, the quality of international migration data is very good, but in some countries it is less reliable. A high-level check of population accounting consistency can be made by comparing past

- recorded net international migration (calculated as immigration minus emigration), with
- residual net migration (calculated as population growth minus recorded natural change).

A more detailed check of accounting consistency involves comparing residual net migration with recorded net migration by sex and cohort for a recent period to ensure that the age profiles of immigration and emigration are reasonable. Does period-cohort

- recorded net migration (immigration minus emigration) equal
- residual net migration (calculated as cohort population change plus period-cohort deaths)?

If there are large discrepancies, then there is a problem. If the population estimates are deemed more reliable than the migration data, then either or both immigration and emigration values should be adjusted.

*Step 3: Prepare assumptions on the expected future direction of fertility, mortality and migration*

Once data quality checks have been completed, and data adjustments made if necessary, then assumptions on the future of fertility, mortality and migration can be prepared. Section 5 provides an overview of various approaches and methods which can be used.

*Step 4: Enter all input data into the projection program/spreadsheet*

The next stage involves loading all the input data into computer code or a workbook for producing population projections. Even with highly automated processes there is still potential for errors to creep in (e.g. using old assumptions; male and female populations being mixed up), so it is useful to double check all the input data has been entered, and entered in the correct locations.

*Step 5: Calculate the projections*

Then the population projections are calculated using the cohort-component model projection equations described next in section 4. The Excel workbook accompanying this paper can be used, or you may wish to write your own code.

*Step 6: Check outputs for plausibility and consistency, and revise assumptions if necessary*

An important part of the projection process is checking that the output projections are plausible and internally consistent. Unfortunately, it is quite possible, even with carefully prepared projection assumptions, for the cohort-component model to generate implausible or suboptimal outcomes. These outcomes can be caused by data quality issues, random noise in input data, errors in projection assumptions, mistakes in loading assumptions data into the projection program/spreadsheet, projection model coding errors, and the lack of mechanisms in the cohort-component model to maintain sensible relationships between certain aspects of the projections. For example, it might be the case that sex ratios become increasingly unlikely over the projection horizon. Section 6 covers some of the key checks which should be made to projections output. A further check on the input assumptions and projection model can be made by creating a test run set

of projections from a few years ago up to the latest population estimate, and comparing the 'projected' populations and components with recorded data.

## 4. The cohort-component model

### 4.1. Overview

The cohort-component model projects the population broken down by cohort by adding and subtracting the demographic components of change. It is usually also disaggregated by sex. The Lexis diagram in Figure 2 illustrates the age-time spaces of the cohort-specific population accounting equation which lies at the heart of the model. The population of a cohort at the start of the time interval,  $t$ , is depleted by deaths and emigration during the interval, but supplemented by immigration. After taking into account these component changes, we have the cohort population size at the end of the time interval,  $t + 1$ .

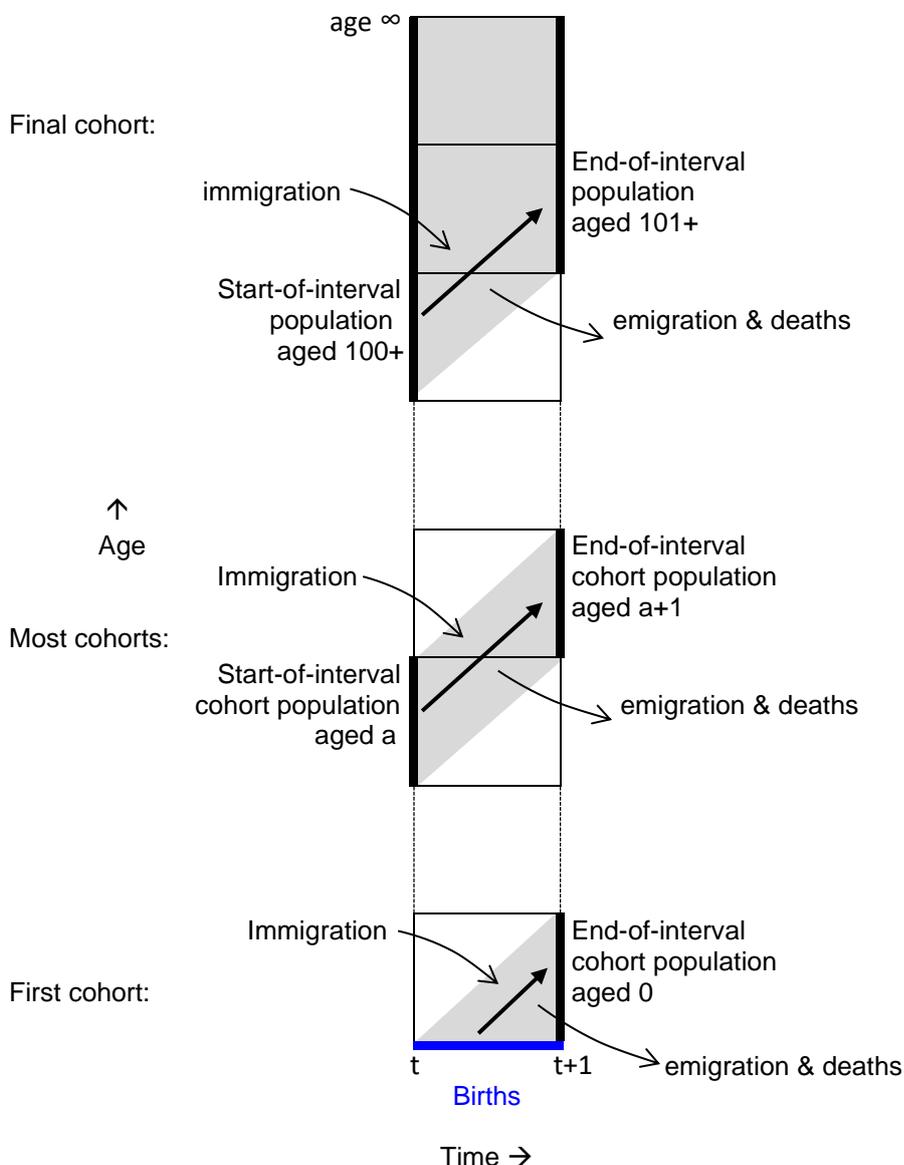


Figure 2: Defining features of the cohort-component population projection model

Notes. Population stocks are shown by thick black lines, and births by the thick blue line. The age-time extent of immigration, emigration and deaths for the cohorts shown is depicted by the grey shaded areas.

The middle part of the diagram shows the age-time spaces of stocks and flows for all cohorts except the very oldest and youngest. The cohort aged  $a$  at the start of the projection interval ages to become aged  $a + 1$  by the end, and during the year experiences population losses through deaths and emigration, and gains from immigration. The lower part of the diagram illustrates the newly-born infant cohort where the ‘starting’ population is the number of births which occur during the year. The upper section of the diagram illustrates the slightly different situation at the highest ages. The elongated extent of this part of the diagram indicates the absence of an upper age limit. So, for example, if the oldest open-ended age group in the projections is 100+, then this part of the diagram illustrates the population aged 100+ at time  $t$  ageing to become the 101+ population at time  $t + 1$ .

There are three main stages to the cohort-component model calculations:

- Stage 1: project cohorts already alive at the start of the projection interval
- Stage 2: project the number of births
- Stage 3: project newly-born babies to age 0 at the end of the projection interval

which we describe in turn below.

#### 4.2. Stage 1: Project existing cohorts

In the first stage, all cohort populations alive at the start of the projection interval are projected. This refers to all cohorts except for the cohort of babies born during the interval. The cohort-specific population accounting equation may be written as:

$$P_{s,a+1}(t + 1) = P_{s,a}(t) - D_{s,a \rightarrow a+1} - E_{s,a \rightarrow a+1} + I_{s,a \rightarrow a+1} \tag{2}$$

where:

$s$  = sex

$a$  = age group

$a + 1$  = age group one year older than  $a$

$a \rightarrow a + 1$  = the period-cohort aged  $a$  at the start of the time interval which shifts to age  $a + 1$  by the end (as shown by the shaded parallelogram in the middle of Figure 2).

Time labels  $t, t + 1$  have been omitted from the components in the equation to avoid clutter.

The accounting equation above is converted to a projection equation by substituting deaths and emigration flows for occurrence/exposure rates multiplied by populations-at-risk (Rees 1984; Willekens & Drewe 1984):

$$P_{s,a+1}(t + 1) = P_{s,a}(t) - d_{s,a \rightarrow a+1} \frac{1}{2} (P_{s,a}(t) + P_{s,a+1}(t + 1)) - e_{s,a \rightarrow a+1} \frac{1}{2} (P_{s,a}(t) + P_{s,a+1}(t + 1)) + I_{s,a \rightarrow a+1} \tag{3}$$

where:

$d$  = death rate

$e$  = emigration rate.

The population-at-risk is the number of person-years the cohort population is exposed to the risk of death or emigration over the projection interval. It is approximated by the mean of the start- and end-of-interval populations. For the moment, we leave aside the issue of preparing the various assumed age-specific rates and return to this in section 5 on input data and assumptions.

In the cohort-component model described here, immigration is projected directly as immigration numbers<sup>3</sup>. It would be possible in theory to project immigration as the product of immigration rates and the rest of the world's population, but we would then have to include the rest of the world in the projections to calculate the required population-at-risk for sending migrants to the country we are producing projections for. In any case, immigration to many countries is strongly shaped by the migration policies and regulations of that country. For this reason, it is often projected directly as immigration flows. Some cohort-component models use net international migration, but we advise against this wherever possible. See Box 2 for a brief note on this issue.

By re-arranging the projection equation, the end-of-interval populations can be removed from the right-hand side (Willekens and Drewe 1984). First, multiplying out the terms in brackets gives:

$$P_{s,a+1}(t+1) = P_{s,a}(t) - \frac{1}{2}d_{s,a \rightarrow a+1} P_{s,a}(t) - \frac{1}{2}d_{s,a \rightarrow a+1} P_{s,a+1}(t+1) \\ - \frac{1}{2}e_{s,a \rightarrow a+1} P_{s,a}(t) - \frac{1}{2}e_{s,a \rightarrow a+1} P_{s,a+1}(t+1) + I_{s,a \rightarrow a+1}.$$

Then those terms involving the end-of-interval populations are all moved to the left-hand side of the equation:

$$P_{s,a}(t+1) + \frac{1}{2}d_{s,a \rightarrow a+1} P_{s,a+1}(t+1) + \frac{1}{2}e_{s,a \rightarrow a+1} P_{s,a+1}(t+1) \\ = P_{s,a}(t) - \frac{1}{2}d_{s,a \rightarrow a+1} P_{s,a}(t) - \frac{1}{2}e_{s,a \rightarrow a+1} P_{s,a}(t) + I_{s,a \rightarrow a+1}.$$

This can be simplified to:

$$\left(1 + \frac{1}{2}d_{s,a \rightarrow a+1} + \frac{1}{2}e_{s,a \rightarrow a+1}\right) P_{s,a+1}(t+1) \\ = \left(1 - \frac{1}{2}d_{s,a \rightarrow a+1} - \frac{1}{2}e_{s,a \rightarrow a+1}\right) P_{s,a}(t) + I_{s,a \rightarrow a+1}$$

and then re-arranged so that just the end-of-interval population remains on the left-hand side of the equation:

$$P_{s,a+1}(t+1) = \frac{\left(1 - \frac{1}{2}d_{s,a \rightarrow a+1} - \frac{1}{2}e_{s,a \rightarrow a+1}\right)}{\left(1 + \frac{1}{2}d_{s,a \rightarrow a+1} + \frac{1}{2}e_{s,a \rightarrow a+1}\right)} P_{s,a}(t) + \frac{I_{s,a \rightarrow a+1}}{\left(1 + \frac{1}{2}d_{s,a \rightarrow a+1} + \frac{1}{2}e_{s,a \rightarrow a+1}\right)} \quad (4)$$

To illustrate the calculations, we use the example of the female cohort aged 45 years in mid-2019 and 46 in mid-2020. If the population in 2019 was 164,940, the period-cohort death rate 0.001127, the emigration rate 0.005916, and the immigration flow 1,693.3, then the projected population in 2020 is:

<sup>3</sup> Alternative ways of handling international migration in the cohort-component are often applied. See also Bijak (2012), Rees (2015), and Vandresse (2015).

$$= \frac{(1 - \frac{1}{2}0.001127 - \frac{1}{2}0.005916)}{(1 + \frac{1}{2}0.001127 + \frac{1}{2}0.005916)} 164,940 + \frac{1,693.3}{(1 + \frac{1}{2}0.001127 + \frac{1}{2}0.005916)}$$

$$= 165,469.8^4$$

### 4.3. Stage 2: Project the number of births

In the second stage, births are projected as the product of age-specific fertility rates and female populations-at-risk, and then summed over all childbearing age groups (usually ages 15 to 49). The total number of projected births is calculated as:

$$B = \sum_a \left( b_a \frac{1}{2} [P_{f,a}(t) + P_{f,a}(t + 1)] \right) \quad (5)$$

where:

$b$  = age-specific fertility rate

$f$  = female population.

Note how the calculations make use of end-of-interval female populations which were calculated in the first stage. Age-specific fertility rates are period-age (shown by the shaded square in Figure 1) and not period-cohort because this is the form in which age-specific births and fertility statistics are often provided<sup>5</sup>.

As an example, consider females aged 30. If the age-specific fertility rate for women aged 30 is 0.112436 and the start- and end-of-interval populations of women aged 30 are 193,557 and 198,656 respectively, then the projected number of births born to women in this age group is:

$$= 0.112436 \frac{1}{2} [193,557 + 198,656]$$

$$= 22,049.4$$

Projected births from each age group of women are summed to obtain total births, and they are then divided into male and female births using the assumed sex ratio at birth:

$$B_f = B \frac{100}{100 + SRB}$$

$$B_m = B \frac{SRB}{100 + SRB}$$

where:

$SRB$  = sex ratio at birth, the number of males born for every 100 females.

Typically, this lies between 103 and 107 male births per 100 female births, with a mean of about 105 (Ritchie 2019).

<sup>4</sup> In reality, there cannot be fractional population numbers, of course. But during the calculations, fractions are usually retained, and rounding applied to the final projection outputs at the end of the calculation process.

<sup>5</sup> It would be possible to project births using period-cohort fertility rates and populations-at-risk, but it isn't necessary for projecting total numbers of births.

As an example, assume the total number of projected births is 304,365 and the SRB is 105.7 males per 100 females. Then the number of female babies is:

$$= 304,365 \frac{100}{100+105.7}$$

$$= 147,965.5$$

and the number of male babies:

$$= 304,365 \frac{105.7}{100+105.7}$$

$$= 156,399.5$$

#### 4.4. Stage 3: Project the infant cohort

The third stage involves projecting the cohort of newly-born babies forwards to obtain the population aged 0 at the end of the projection interval. The population accounting equation includes the number of births as the 'initial' population:

$$P_{s,0}(t+1) = B_s - D_{s,b \rightarrow 0} - E_{s,b \rightarrow 0} + I_{s,b \rightarrow 0} \quad (6)$$

where:

$b \rightarrow 0$  = the infant cohort born during the projection interval and aged 0 at the end (illustrated by the shaded triangle at the bottom of Figure 2).

For the projection equation, deaths and emigration flows are replaced by rates multiplied by the population-at-risk:

$$P_{s,0}(t+1) = B_s - d_{s,b \rightarrow 0} \frac{1}{2} P_{s,0}(t+1) - e_{s,b \rightarrow 0} \frac{1}{2} P_{s,0}(t+1) + I_{s,b \rightarrow 0} \quad (7)$$

The population-at-risk is approximated as half the end-of-interval population. This provides a reasonable estimate of the person-years lived by the newly-born cohort where there is low infant mortality because the end-of-interval population consists of nearly all members of the new cohort who were born, and they were part of the population for an average of half the time interval.

With re-arranging, this projection equation can be simplified to:

$$P_{s,0}(t+1) + \frac{1}{2} d_{s,b \rightarrow 0} P_{s,0}(t+1) + \frac{1}{2} e_{s,b \rightarrow 0} P_{s,0}(t+1) = B_s + I_{s,b \rightarrow 0}$$

then:

$$\left(1 + \frac{1}{2} d_{s,b \rightarrow 0} + \frac{1}{2} e_{s,b \rightarrow 0}\right) P_{s,0}(t+1) = B_s + I_{s,b \rightarrow 0}$$

and:

$$P_{s,0}(t+1) = \frac{1}{\left(1 + \frac{1}{2} d_{s,b \rightarrow 0} + \frac{1}{2} e_{s,b \rightarrow 0}\right)} B_s + \frac{I_{s,b \rightarrow 0}}{\left(1 + \frac{1}{2} d_{s,b \rightarrow 0} + \frac{1}{2} e_{s,b \rightarrow 0}\right)} \quad (8)$$

To illustrate the calculations, we take the female cohort born during 2019-20 which ends the projection interval in mid-2020 aged 0. If there are 147,965.5 births, a death rate of 0.005207, an

emigration rate of 0.012508, and an immigration flow of 1,865.8, then the end-of-interval population aged 0 is:

$$= \frac{1}{\left(1 + \frac{1}{2} 0.005207 + \frac{1}{2} 0.012508\right)} 147,965.5 + \frac{1,865.8}{\left(1 + \frac{1}{2} 0.005207 + \frac{1}{2} 0.012508\right)}$$

$$= 148,515.8$$

#### 4.5. Prepare for the next projection interval

The end-of-interval populations then become the start-of-interval populations for the next projection interval. This requires summing the end-of-interval populations for the two oldest cohorts. For example, if the oldest age group in the projections output is 100+, then the projected end-of-interval populations at age 100 and age 101+ must be summed to obtain the 100+ start-of-interval population for the next projection interval.

$$P_{s,100+}(t + 1) = P_{s,100}(t + 1) + P_{s,101+}(t + 1) \quad (9)$$

The projection calculations are then repeated for as many years ahead as required.

#### Box 2: Why is it best to avoid projecting international migration as net migration?

Projecting international migration as net migration numbers is appealing because the calculations are simple and there are minimal data requirements. In fact, net migration can be calculated from a recent year as the remaining cohort population change once deaths have been taken into account, so no actual migration data are needed. However, there are both conceptual and practical limitations of modelling net migration. Conceptually, there is no actual demographic process of net migration, only immigration flows into a country and emigration flows out. In addition, by using net migration numbers, we remove the direct relationship between the age pattern of migration and the population age structure. By using emigration rates, emigration is projected as a function of the population-at-risk in each period-cohort, so that the age pattern of emigration is influenced by population age structure. From a practical perspective, net migration age patterns are much more variable over time than immigration and emigration age patterns, and are shaped by the relative amounts of immigration and emigration. It is therefore difficult to adjust the age profile of net migration over the course of the projection horizon (in response to migration policy changes or global pandemics, for example). Simply scaling the net migration age profile up and down is unlikely to give a realistic outcome. In addition, in some extreme circumstances, negative net migration held constant over long periods can yield 'negative' populations.

#### 4.6. Strengths and weaknesses

The cohort-component model is widely seen as the 'gold standard' method of population projection, and it possesses many excellent features. But it also has some weaknesses which are worth keeping in mind. Some of the strengths and weaknesses are listed in Table 1 below (see also Burch 2018).

Table 1: Some of the strengths and weaknesses of the cohort-component model

Strengths	Weaknesses
Good representation of actual population processes. It includes the effects of population age structure and age-specific rates	Assumes a homogenous population, with no allowance for the changing internal composition
Can be accurate if good assumptions about fertility, mortality and migration are chosen	Can be highly inaccurate if poor assumptions about fertility, mortality and migration are chosen
Provides age-disaggregated projections which are useful for a wide variety of planning, policy, budgeting, and service delivery purposes	Requires a non-trivial amount of input data; and assumptions about future fertility, mortality and migration are not intrinsic to the model and have to be prepared separately
Provides the starting point for many derived projections (e.g., labour force; living arrangement; school enrolments)	Lack of in-built mechanism for ensuring consistency between male and female projections
Useful for understanding population change (e.g., by creating alternative future scenarios; or applying to past periods)	Immigrants to the population are assumed to take on the fertility, mortality and emigration rates assumed for the whole population
Simple mathematics. Basic versions can be implemented in a spreadsheet	Births are produced by women only, rather than men and women.

#### 4.6. The Excel workbook

The Excel workbook accompanying this paper (National projection model example.xlsx) contains an example set of projections for Australia using the cohort-component model equations presented above. It includes the following worksheets.

**Notes:** A short introduction to the layout and workings of the Excel workbook.

**JumpOff:** The jump-off populations should be supplied in this sheet.

**Fertility:** This is where the fertility assumptions are placed. To aid assumption-setting, fertility assumptions are provided separately for the level (TFR) and age profiles (age-specific rates) of fertility. The age-specific fertility rates are scaled to match the TFR further down the sheet. This approach allows you to make slight adjustments to the TFR assumptions without having to re-calculate the age-specific rates.

**Mortality:** This is where the projected  ${}_1L_x$  life table values, used to calculate death rates by period-cohort, are placed.

**Migration:** Like fertility, migration assumptions are split into level and age profile components to aid assumption-setting. For immigration, both the annual total immigration intake and immigration numbers by period-cohort are required. For emigration, the assumed Gross Migraproduction Rate (GMR) (Rogers et al. 1978) for the overall level of emigration, and emigration rates by period-cohort, are needed. The GMR is simply the sum of all age-specific rates, much like the Total Fertility Rate. Further down the sheet the supplied immigration numbers and emigration rates are scaled to sum to, respectively, the total immigration and emigration GMR assumptions.

**Projections:** The projection calculations are carried out in this sheet using equations 4, 5 and 8. Scroll down to find projected births, deaths, and emigration flows by sex and period-cohort.

**Population:** The final projected populations are set out in this sheet.

**AccountsSummary:** Summary population accounts for each single year projection interval are provided in this sheet (start-of-interval populations, births, deaths, immigration, emigration, and end-of-interval populations).

## 5. Input data and projection assumptions

The cohort-component model requires the following input data:

- Jump-off populations by sex and age group
- Future fertility rates by age of mother
- Future mortality rates by sex and period-cohort
- Future emigration rates by sex and period-cohort
- Future immigration numbers by sex and period-cohort

### 5.1. Jump-off populations

Jump-off population estimates by sex and single years of age are needed to provide the starting point of the projections. Ideally the age disaggregation would extend up to an age high enough so that there are relatively few people in the highest open-ended age group. Usually the jump-off populations would be the most recently published population estimates available from the national statistical office.

### 5.2. Fertility

Age-specific fertility rates are required for each year of the projection horizon. Sometimes age-specific rates are projected separately from the Total Fertility Rate, and then adjusted to be consistent with the TFR if necessary. Three broad approaches are suggested here:

- 1) Assume the average TFR and age-specific fertility rates of recent years remain constant into the future. If fertility has remained approximately constant in recent years, and there are no obvious reasons for assuming change, this could be a reasonable assumption.
- 2) Use fertility assumptions prepared by others, such as the national statistical office (e.g. ABS 2018), the United Nations Population Division (2019), or researchers who have prepared projections and make the data available (e.g. McDonald 2020).
- 3) Prepare your own projections. This option does come with some challenges, however. There is no single fertility projection method widely regarded as the best option. And, despite considerable amounts of research over many decades to understand fertility better, it remains notoriously hard to predict.

Some fertility projections are produced from extrapolative models, some from explanatory models linked to variables such as unemployment rates, some projections are created by trending from recent fertility to a long-run target, while others are based on expert opinions. An overview of fertility projection methods used in practice can be found in Gleditsch and Syse (2020), while reviews

of the academic literature are included in Booth (2006), Shang and Booth (2020), Hilton et al. (2019), and Bohk-Ewald et al. (2018), though this last paper focuses largely on cohort fertility projections. In practice, fertility assumptions are often created by drawing on a variety of models and approaches.

The TFR assumptions for Australia in the example Excel workbook accompanying this paper were prepared qualitatively, but were informed by other projections (McDonald 2020), research on the impacts of major economic shocks to fertility (e.g. Matysiak et al. 2020; Sobotka et al. 2011), and the declining fertility trend observed in Australia in recent years (ABS 2020b). A long-run TFR of 1.70 was assumed, with a short-run dip to allow for the economic impacts of COVID-19. The age profile of fertility rates was projected by extrapolating the parameters of the Peristera and Kostaki (2007) model of fertility, which fits well to the Australian fertility rate age profile.

### 5.3. Mortality

Death rates by sex and period-cohort are required for every year of the projection horizon. Unlike fertility, the assumption of holding rates constant into the future is rarely a good choice for mortality because it tends to decline over time. Therefore, two broad approaches are recommended here.

- 1) Use mortality assumptions prepared by others, such a national statistical office.
- 2) Prepare your own mortality projections. National mortality forecasting has received the greatest amount of attention in demographic forecasting research and there is no shortage of methods (Booth and Tickle 2008; Ediev 2020; Terblanche 2015) and software (e.g., Hyndman et al. 2019; Sevcikova et al. 2019) to choose from.

Generally, mortality trends are fairly stable over time, making it easier to forecast than fertility and migration (though in some countries COVID-19 has now interrupted those trends). Many of the mortality forecasting methods and software packages devised over the last couple of decades have been shown to produce quite accurate forecasts (e.g. Terblanche 2016).

The mortality assumptions in the Excel workbook accompanying this paper were created using Ediev's (2008) extrapolative mortality model. Age-specific death rates (in period-age form) were projected, and then life tables created. The Excel workbook contains projected  ${}_1L_x$  values which are used to calculate period-cohort death rates. Because these values comprise the life table stationary population, the difference between populations at adjacent ages can be interpreted as the number of period-cohort deaths, shown by the shaded parallelogram in the Lexis diagram in Figure 3. Note that the life table populations and deaths exactly match the age-time locations shown for the cohort-component model earlier (middle of Figure 2). Death rates are calculated with these deaths in the numerator and the mean of the two populations as the population-at-risk:

$$d_{a \rightarrow a+1} = \frac{{}_1L_x - {}_1L_{x+1}}{\frac{1}{2}({}_1L_x + {}_1L_{x+1})}$$

where  $x$  is the exact starting age of age group  $a$ . The sex subscript is omitted in the equation to prevent clutter.

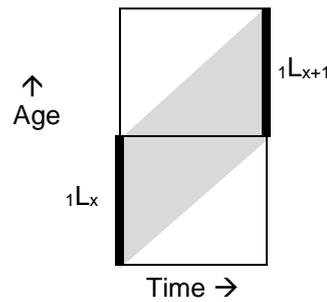


Figure 3: Life table data used to calculate period-cohort death rates

For newly-born infants and at the very highest ages, death rates are calculated slightly differently:

$$d_{b \rightarrow 0} = \frac{l_0 - {}_1L_0}{\frac{1}{2} {}_1L_0}$$

$$d_{100+ \rightarrow 101+} = \frac{{}_\infty L_{100} - {}_\infty L_{101}}{\frac{1}{2}({}_\infty L_{100} + {}_\infty L_{101})}$$

where

$l_0$  = the radix (or number of births) in the life table population.

These deaths and populations exactly match the age-time locations as indicated for the youngest and oldest cohorts in Figure 2.

### 5.4. International migration

International migration is hard to forecast. It tends to exhibit quite volatile trends in volume over time in response to changes of policy (at least for immigration), macroeconomic conditions, and major world events. And for some countries (though not Australia), international migration data suffers from quality and coverage problems. Fortunately, the age profiles of immigration and emigration rates tend to be far more stable. Our suggested approaches to migration forecasting are like those for fertility:

- 1) Assume the trends of recent years remain constant into the future. In the absence of clear changes in trends in immigration and emigration, this can be a reasonable assumption.
- 2) Use migration assumptions prepared by others (e.g. ABS 2018).
- 3) Prepare your own migration projections. This can be a challenging task.

Unlike fertility and mortality, there are relatively few available projection methods and software packages to choose from and, unfortunately, no single method has been found to produce consistently accurate migration projections. Projection approaches include extrapolative models, explanatory models, trending to a target, expert panels, and what might best be described as a “forecaster’s judgement” approach that is not based on any formal model or process. This last approach qualitatively takes into account recent migration trends, perceptions of factors driving those trends, broad economic trends, policy influences (including immigration program plans), and expert opinions, among other things. It is only loosely connected to theory and modelling – but that is not necessarily a weakness given that theory and modelling have not yet delivered a robust migration projection method.

If a more formal process or model is to be used, what are the options? A few examples are given here. One option is to apply time series or econometric models. Examples of these types of models can be found in Bijak et al. (2019), de Beer (2008), and Raymer and Wiśniowski (2018). Statistics Norway uses a model based on neoclassical economic theory to project various immigration streams (Cappelen et al. 2015). Explanatory variables include relative income, Norwegian unemployment rates, and the size of the migrant population. A similar explanatory model of migration has been devised for use in Belgium's population projections (Vandresse 2015). Another approach is to engage expert panels who are actively involved in the assumption formulation process (e.g. Dion et al. 2020; Lutz 2009). The expert panel considers and discusses a wide range of factors which might increase or decrease migration (and other demographic variables) and then suggests future migration levels or distributions. Whichever methods and approaches are selected, many authors have recommended dividing international migration flows into different streams (e.g. permanent immigrants, temporary workers, foreign students, etc.) and creating projections by focusing on the different drivers of those flows (e.g. de Beer 2008; Wilson 2017a).

For the Excel workbook, immigration and emigration assumptions for Australia were prepared separately for the headline indicators of total immigration flows and the emigration Gross Migraproduction Rate (GMR), and the age profiles of immigration and emigration rates. Given that the shapes of immigration and emigration rate age profiles tend to change slowly over time, they were assumed to remain unchanged in the projections. In contrast, the total volumes of immigration and emigration can change markedly from year to year. Given the disruption to international migration created by the COVID-19 pandemic and the closure of the Australian border, we took note of the projected migration flows from the federal government's Centre for Population in formulating our assumptions over the next few years (Centre for Population, 2020). A large drop in both immigration and emigration was assumed in the short term, reflecting the border restrictions introduced in response to the pandemic. For further into the future, we assumed total immigration would resume its long-run gradual upward trend and that the emigration GMR would return to its previous level and remain constant.

## 6. Checking outputs

The final part of the projections process involves checking the projections output for plausibility, consistency, and any possible errors with input data preparation or projections calculations. This is a form of exploratory data analysis for projections data. What constitutes 'plausible' is not easy to define, but projections outputs should look roughly in line with expectations given the chosen input assumptions. Checks of plausibility may not provide a precise answer. Instead, they may only suggest that assumptions would benefit from some refinement, or that certain data inputs may need attention. Unfortunately, just because projections seem plausible does not mean that they will turn out to be accurate, but it should at least mean they will be free of obvious errors or unbelievable assumptions.

**Total population.** Check that the projected total population trend looks plausible in light of historical data by creating a graph of past and projected total populations (and/or total population growth rates). Figure 4 shows an example. In most circumstances there should be no discontinuity or sudden

changes in trend, unless there is clearly good reason for it (e.g., the impact of a pandemic; a sudden immigration policy change).

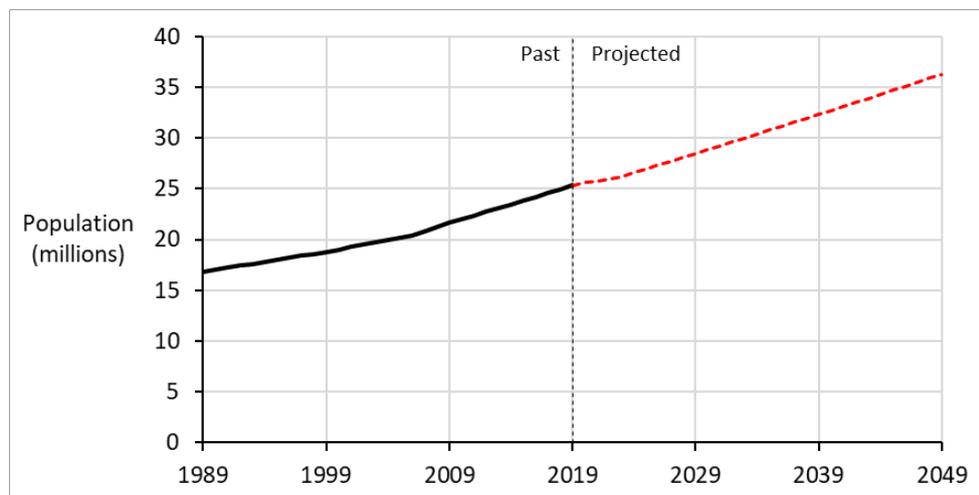


Figure 4: Example graph of total population to check projection outputs

Source: ABS (2020a); authors' projections

**Demographic components.** Do the projected demographic components of change look plausible in light of historical trends? Again, it is useful to create a graph showing past and projected births, deaths, immigration and emigration. Births and deaths normally show relatively smooth trends over time, while immigration and emigration are often more variable.

**Population age-sex structure.** Does the evolution of the age-sex structure of the population look plausible? It is useful to create population pyramids, or at least line graphs of the population by age, over time. If a population has been ageing in the past, it is likely to continue to age in the future. Does the number of 0 year olds seem consistent with the fertility assumptions? Does the sex ratio of the total population (males per 100 females) change slowly over time and not differ from the initial value too much? Does the sex ratio by age show a pattern of values above 100 in the childhood and younger adult ages followed by declines with increasing age? Does it change slowly over time? If male and female life expectancy is projected to converge in the future, does the sex ratio at the older ages increase? Does the relative size of the young adult population seem about right when considering the migration assumptions?

**Others' projections.** It can be useful to compare your projections with those produced by others, such as projections produced by a national statistical office. Unless your projections have radically different assumptions, the broad shape of the outputs is likely to be similar, at least in the initial years of the projection horizon. But differences with other projections should not necessarily be interpreted as problems with your projection. If you have a higher fertility assumption, for example, you would expect a larger projected childhood population.

For a more detailed checklist see 'A checklist for reviewing draft population projections' (Wilson 2017b).

## 7. Conclusions

This paper has presented a brief guide to producing a national population projection with a cohort-component model. Several stages in the projections preparation process were described, starting with data gathering and assumptions preparation, and finishing with projection calculations and output checking. We described the cohort-component model from a population accounting perspective, which derives projection equations directly from simple cohort-specific population accounting equations. And we created an Excel workbook to accompany this paper which implements the cohort-component model for Australia.

The paper covered many of the key elements of the projections preparation process, but it has not been possible to discuss all relevant issues in one short paper. We said little about the smoothing of age profiles of fertility, mortality and migration rates to remove noise, though noise is generally minimal at the national scale. This topic is covered by Preston et al. (2001: chapter 9), de Beer (2011, 2012), and Wilson (2020). We didn't go into much detail about population accounting reconciliation because accounting discrepancies are usually not a major problem at the national scale, nor did we consider forecast uncertainty and error (on this, see Keilman 2018, 2020, and for an Australian example of probabilistic forecasts, see Bell et al. 2011). We also did not cover any topics which become relevant once the projections have been finalised, such as the best ways of communicating projections to stakeholders (Johnstone 2015).

It is important to note that while preparing a population projection may initially seem like a mostly quantitative and objective exercise, there are actually many judgements which must be made as part of the process. While some may seem like minor technical decisions, they require careful consideration because they might substantially impact the projection outputs. For example, decisions are made about input data adjustments, about which fertility, mortality, and migration projection methods and assumptions to use, and whether the outputs seem plausible and reasonable. Mastering the calculations for population projections is relatively easy; making good decisions throughout the projections process is more challenging and is developed with practice.

### Excel workbook

The Excel workbook containing a cohort-component model and example population projections for Australia may be downloaded from <https://doi.org/10.6084/m9.figshare.13565222.v3>.

### Acknowledgements

Funding support for TW from the Australian Research Council Centre of Excellence in Population Ageing Research is gratefully acknowledged (project number CE1101029).

Helpful comments on an earlier draft of this paper were kindly provided by Ahana Banerjee Sen, Rosabella Borsellino, Dagmara Laukova, and the anonymous reviewers.

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